



# Image-Reject and Single-Sideband Mixers

Image-reject mixers (IRMs) and single-sideband mixers (SSMs) play a key role in many of today's microwave and rf systems [1, 2, 3]. IRMs and SSMs reduce system cost and complexity by removing the need for expensive preselection, and one or more stages of up- or downconversion. IRMs simplify downconversion by employing phase-cancellation techniques to separate the downconverted products resulting from the undesired image and desired rf inputs. Similarly, SSMs simplify up-conversion by separating the upconverted lower sideband (LSB) from the upconverted upper sideband (USB). In both IRMs and SSMs, two mixing products are separated and channelized into two different output ports to be further processed or terminated.

This article provides a working knowledge of present IRM and SSM technology. It gives an overview of what these devices do, how they operate, and some practical performance considerations. In addition, two appendices are given: one that provides a simplified analysis procedure for evaluating quadrature-mixer circuits, and another that correlates image rejection and sideband suppression with circuit parameters.

## THE IMAGE

The image is an unwanted input signal to the mixer. Its frequency is above or below the local oscillator (LO) frequency by an amount equal to the IF frequency. For example, in Figure 1, if  $f_{R1}$  is the desired if input signal, then  $f_{R2}$  is its image. The image and desired inputs both mix with the LO and downconvert to the same frequency. This poses a problem in conventional DB (double-balanced) mixers because the two downconverted products interfere with each other, since they exit at the IF port together. IRMs avoid this problem by channelizing the two products into separate output ports.

## CONVENTIONAL MIXERS

Conventional double-balanced mixers use filters to block the image from entering the mixer, so that no down-converted image is allowed to be generated by the mixer. Since the desired and image signals are always separated in frequency by twice the IF frequency, the IF frequency must be high enough to allow the preselector in front of the mixer to block the image, but still allow the desired if signal to enter the mixer. As the IF frequency is reduced, the desired and image signals move closer together in frequency (converging on  $f_L$ ), forcing the selectivity of the preselector to increase in order to separate the two adjacent input signals. Preselector complexity also increases for tunable receivers because the preselector must track with the LO frequency, to maintain the normally constant IF output frequency.

Also since the IF frequency must be relatively

high to simplify preselection, a number of downconversion stages are required to down-convert the rf input to the baseband frequency for detection.

## IRMS AND SSMS

In comparison to conventional DB mixers, IRMs achieve image-rejection through phase cancellation, not filtering, so the frequency spacing between the image and desired inputs can be negligible. This means that downconversion can be accomplished without preselection, and in fewer stages, saving the cost of extra mixers, amplifiers, local oscillators, and fitters. For similar reasons, upconversion can also be simplified by using single-sideband mixers.

## WHAT IRMS AND SSMS DO

Figure 2 shows the circuit configuration used for image-reject mixers and single-sideband

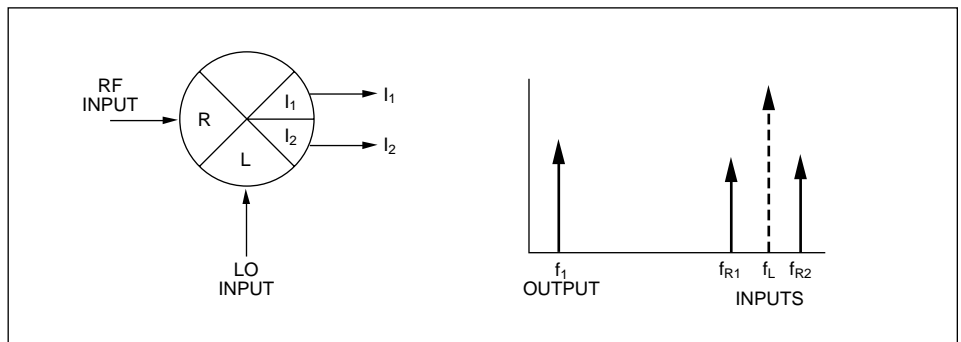


Figure 1. IRM application.

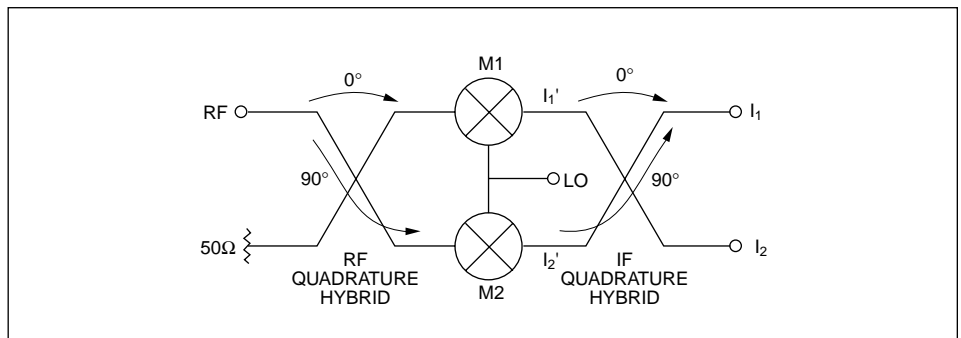


Figure 2. Block diagram of an image-reject and single-sideband mixer.



mixers. The only differences between them are their respective applications and parameters.

### IRM

Figure 1 shows how the circuit of Figure 2 is operated as an IRM. The signal at  $f_{R1}$  will downconvert to exit at  $I_1$ , and the signal at  $f_{R2}$  will down-convert to exit at  $I_2$ . If  $f_{R1}$  is the desired signal, then  $f_{R2}$  is its image.

Ideally, none of the downconverted image signal exits the desired IF output port. However, since amplitude and phase imbalances exist in practical circuits, some of the downconverted image will be present at the desired IF output port. Image rejection is defined as the ratio of the downconverted image signal power exiting the desired IF port, to that of the desired signal, exiting the same IF output port. For example, if the downconverted image and desired signal levels at  $I_1$  are -30 dBm and -10 dBm respectively, then the image rejection is 20 dB. Good image rejection requires close amplitude and phase matching, low mixer VSWR, and directivity.

### SSM

Figure 3 shows how the circuit of Figure 2 is operated as a single-sideband mixer. The SSM provides a single-sideband suppressed carrier output. A LSB or USB output can be selected by choosing which I port to drive with the IF signal. An IF into  $I_1$ , results in an LSB output, and an input into  $I_2$ , results in a USB output. SSMs have two main parameters: sideband suppression and carrier suppression. Sideband suppression is analogous to image rejection, and is defined as the ratio of the undesired sideband signal power to that of the desired sideband signal power at the if output port. Carrier suppression is a measure of how much of the carrier signal leaks through the SSM to become present at the rf output, and is defined as the ratio of the carrier-power level at the output port to that of the desired output-power level at the if output port.

Both sideband and carrier suppression are specified in terms of dBc because they are

measured with respect to the desired LSB or USB output-power level.

### HOW IRMS AND SSMS OPERATE

Image-reject and single-sideband mixer operation may be explained as follows: In any mixer, the phase angles of its rf and LO input signals are conserved through the mixing process, so that the phase of the IF output equals the sum of the if and LO input phase angles, multiplied by their respective harmonic coefficients, m and n. These coefficients define the inter-modulation products exiting the mixer  $f_{IM} = mf_R + nf_L$ , where m and n are positive or negative integers. For the desired and image downconverted products, m and n equal  $\pm 1$ . For example, referring to Figure 1, if the frequency of the downconverted desired signal is  $f_{IF} = f_L - f_{R1}$ , then  $n = 1$  and  $m = -1$ , and its phase angle will equal  $(\theta_L - \theta_{R1})$ , where  $\theta_L$  and  $\theta_{R1}$  are the phase angles of the LO and rf inputs, respectively. Similarly, the frequency of the downconverted image signal is  $f_{IM} = f_{R2} - f_L$  so that  $m = 1$  and  $n = -1$ , and its phase angle equals  $(\theta_{R2} - \theta_L)$ .

Figure 2 shows that both IRMs and SSMS comprise two mixers, two quadrature power dividers, and one in-phase power divider. These are all passive devices, and can act together to significantly enhance system cost-effectiveness, performance, and reliability. Mixers M1 and M2 have IF output currents,

$I_1'$  and  $I_2'$ , respectively. The phase angles of  $I_1'$  and  $I_2'$  are  $0^\circ$  and  $90^\circ$ , respectively. For both mixers,  $(\theta_L$  is set equal to zero because the LO is applied in-phase to M1 and M2. Also, since the IF inputs to M1 and M2 are in quadrature; i.e.,  $90^\circ$  out of phase with respect to each other,  $(\theta_R$  for M1 is set equal to zero, and  $(\theta_R$  for M2 is set equal to  $90^\circ$ . Hence,  $I_1' = I_{mn} \angle 0^\circ$  and  $I_2' = I_{mn} \angle m90^\circ$ .  $I_{mn}$  is the same for M1 and M2 because the two mixers are assumed to have matching conversion-loss characteristics.

$I_1'$  and  $I_2'$  combine in the output quadrature power divider in such a way as to channelize the  $(f_L - f_{R1})$  product into output port  $I_1$ , and the  $(f_{R2} - f_L)$  product into output port  $I_2$ . When downconverting, one product is taken to be the desired output, and the other is taken to be the image output, which is terminated. The following shows how this channelization occurs:

$$I_1 = 1/2 (I_1' + I_2' \angle 90^\circ) = 1/2 (I_{mn} + I_{mn} \angle (m + 1) 90^\circ) = \begin{cases} I_{mn} & ; m = -1(f_L - f_{R1}) \\ 0 & ; m = 1(f_{R2} - f_L) \end{cases} \quad (1)$$

$$I_2 = 1/2 (I_1' \angle 90^\circ + I_2') = 1/2 (I_{mn} \angle 90^\circ + I_{mn} \angle m90^\circ) = \begin{cases} 0 & ; m = -1(f_L - f_{R1}) \\ I_{mn} \angle 90^\circ & ; m = 1(f_{R2} - f_L) \end{cases} \quad (2)$$

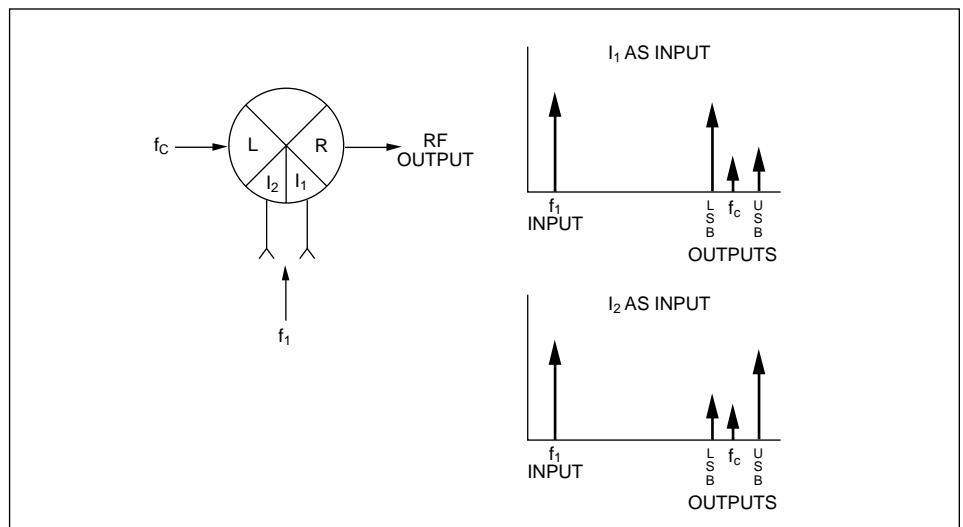


Figure 3. SSM application.



When upconverting,  $I_1$  and  $R_1$  are interchanged, as are  $I_2$  and  $R_2$ , so that the inputs to the mixer are a low-frequency signal injected at  $I_1$  and  $I_2$ , and a microwave carrier injected at the LO port. The outputs are the LSB ( $f_L - f_{IF}$ ) product that exits at  $R_1$ , and the USB ( $f_L + f_{IF}$ ) product that exits at  $R_2$ .

### PRACTICAL PERFORMANCE CONSIDERATIONS

The conversion loss of an IRM includes the losses due to the quadrature hybrids and in-phase power splitter, in addition to the mixer conversion loss. This additional circuitry increases the conversion loss, but not to unacceptable levels. Figure 4 shows the typical conversion loss of an IRM and a SSM. Typical conversion loss is 8.0 dB from 8 to 18 GHz.

The amount of image rejection obtained with an IRM is determined by the circuit amplitude and phase balance. Since circuitry imbalances are frequency dependent, image rejection is also frequency dependent. Figure 5 shows the frequency dependence of the image rejection for the IRM and the sideband suppression for the SSM, typical performance being 22 dB.

Intermodulation products are more critical for the SSM, since there are several spurious products close to the desired output [4]. Suppression of the carrier signal, at frequency  $f_c$ , is also important. Figure 6 shows the typical output spectra for two different applications of the SSM.

Figure 6A shows the SSM using the low-frequency  $f_{IF}$  signal as the high-power input, and Figure 6B shows the SSM with the high-frequency  $f_c$  signal as the high-power input. Inspection of Figure 6 reveals the trade-off between the two different applications. A high-level  $f_c$  signal provides good inter-modulation suppression, but poor carrier suppression; whereas, a high-level  $f_{IF}$  signal provides good carrier suppression at the expense of reduced intermodulation suppression. The

carrier suppression is determined by the mixer L-R isolation.

Also, suppression of the  $1 \times 3$  product is not the same for  $f_c - 3f_i$ , as it is for  $f_c + 3f_i$ . The

$1 \times 3$  product closest to the desired output is always suppressed more than the  $1 \times 3$  product closest to the undesired output. This additional suppression is caused by the input

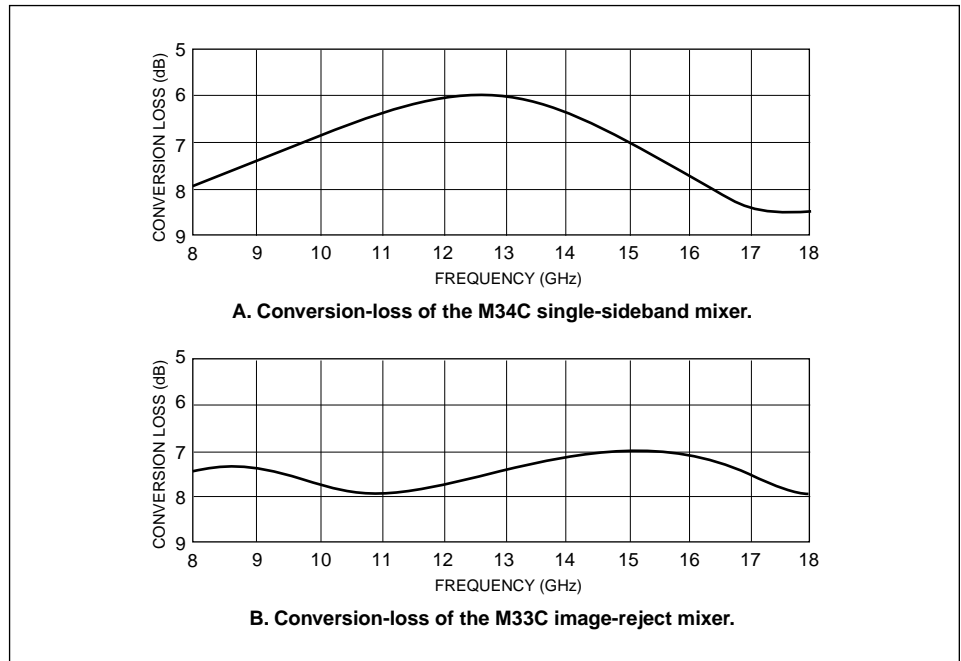


Figure 4. Conversion-loss.

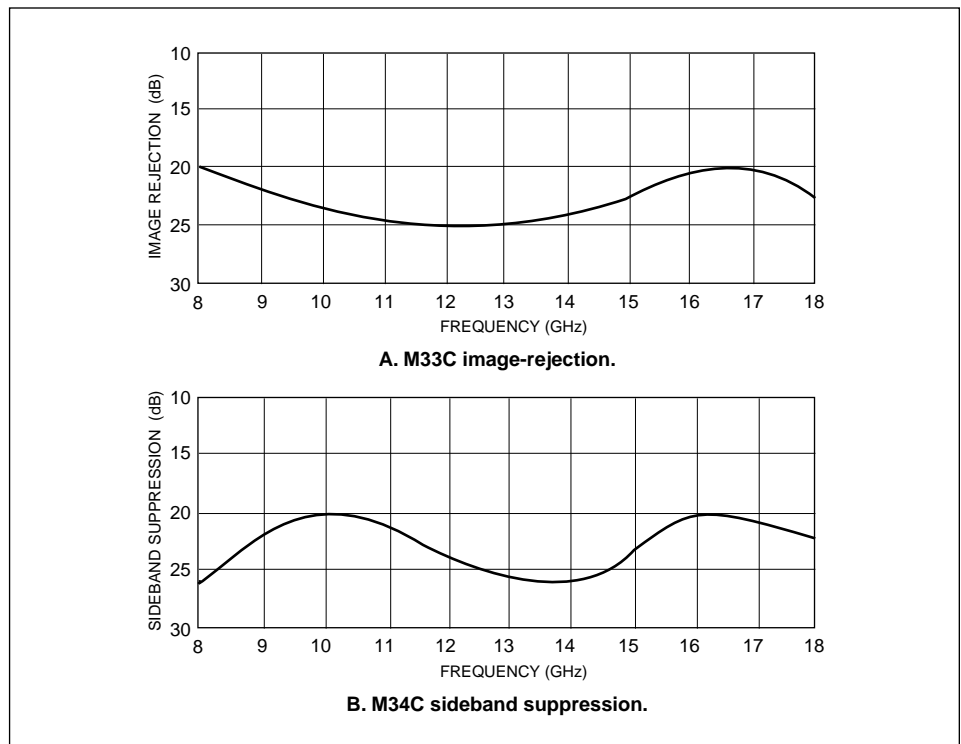


Figure 5. Image-rejection and sideband suppression.



and output quadrature couplers. In fact, every other odd harmonic from the undesired side-band exhibits this characteristic.

### APPENDIX A: SIMPLIFIED ANALYSIS OF QUADRATURE MIXERS

This analysis shows which mixing products will exit the various ports of a quadrature mixer. Much of this analysis has already been published [1, 5], but without the mathematical simplifications included here. The

approach is to determine the Fourier series for the current in each mixer diode, then sum these currents to determine which mixing products exit the various ports.

For example, the IRM of Figure 7 is analyzed. The current in each diode is assumed to flow from anode to cathode, and is written as a double Fourier series:

$$i_D = \sum_{n=1}^{\infty} \sum_{m=1}^{\infty} g_n v_m e^{j(n\omega_L + m\omega_r)t} e^{jm\phi} e^{jn\theta} = k_{nm} e^{jm\phi} e^{jn\theta}$$

This double series results from multiplying

the diode conductance waveform, which is governed by the LO signal, by the waveform for the voltage across the diode, which is governed by the if signal. The amplitude portion of the Fourier series can be reduced to  $K_{nm}$ , or  $K$  for short, since we are only concerned with phase.

The phase angle  $\theta$  corresponds to the difference in phase between the LO input and each of the diode currents. The phase angle  $\phi$  corresponds to the difference in phase between the rf input and each of the diode currents. Four assumptions are made in this analysis:

1. Perfect circuit balance and perfect quadrature couplers
2. Identical diodes
3. Large-signal LO
4. Small-signal rf.

The current in diode 1 can be written as:

$$i_1 = K e^{jm\pi} = K (-1)^m$$

$\phi$  equals  $\pi$  because the if signal is  $180^\circ$  out of

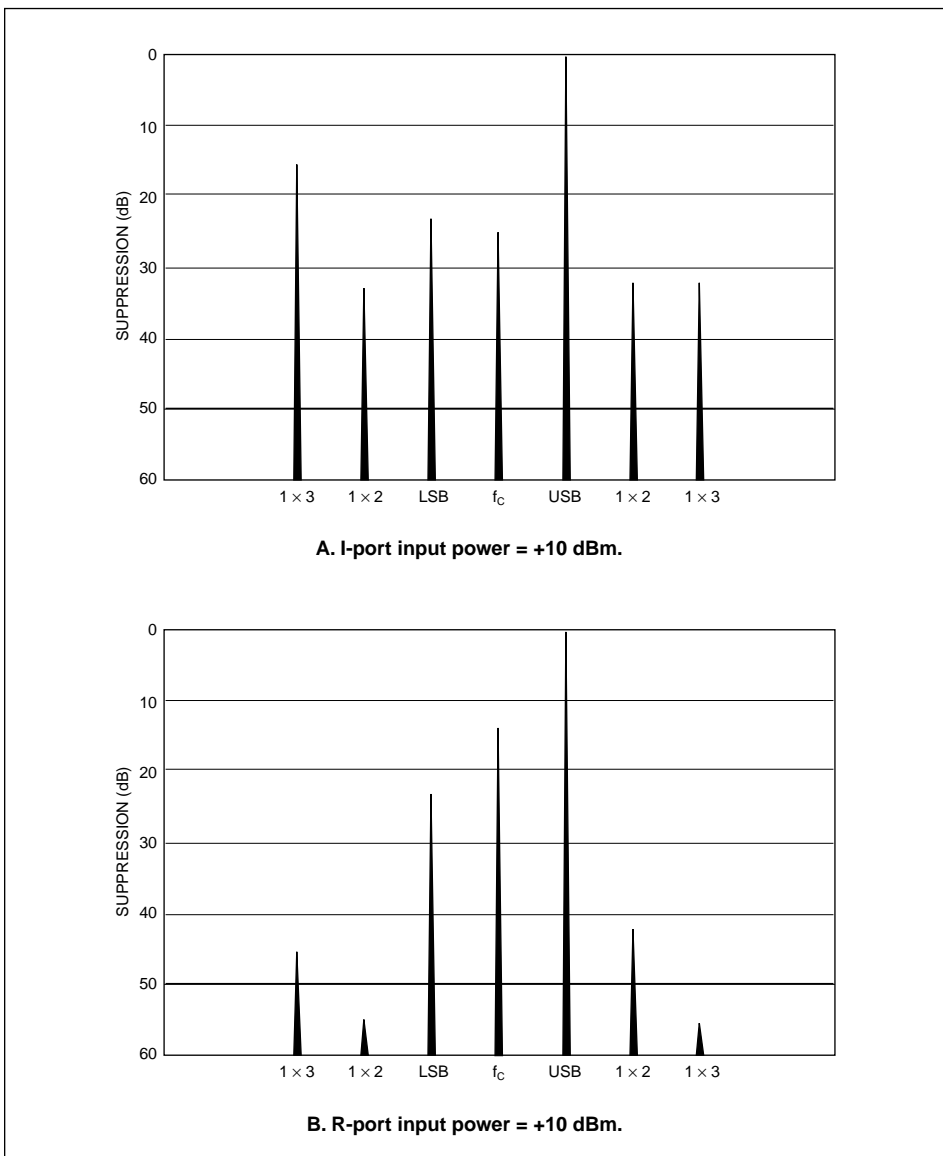


Figure 6. Intermodulation suppression.

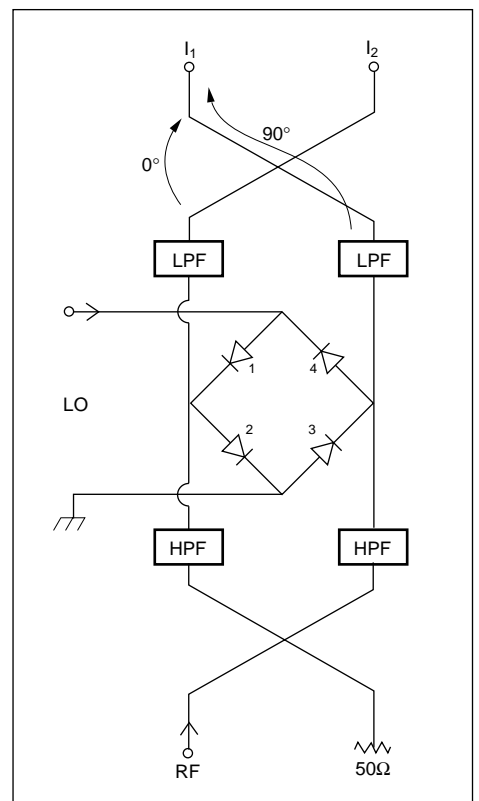


Figure 7. Schematic diagram of the image-reject mixer.



phase with the assumed direction of current-flow in diode 1 (anode-to-cathode).  $\theta$  equals zero because the LO signal is in phase with the current-flow in diode 1. The current in diode 2 can be written as:

$$i_2 = K$$

Both  $\phi$  and  $\theta$  equal zero because the rf and LO inputs are in phase with the current-flow in diode 2. The current in diode 3 can be written as:

$$i_3 = K e^{jm(\pi/2+\pi)} e^{jn\pi} = Kj^{(-m)} (-1)^n$$

$\phi$  equals  $(\pi/2 + \pi)$ : The  $\pi/2$  comes from the rf quadrature-hybrid, and the  $\theta$  comes from the current flow in diode 3 being  $180^\circ$  out of phase with the rf signal exiting the hybrid.

The current in diode 4 can be written as:

$$i_4 = K e^{jm\pi/2} e^{jn\pi} = Kj^m (-1)^n$$

$\phi$  equals  $\pi/2$  because of the rf quadrature hybrid, and  $\theta$  equals  $\pi$  because the LO signal is  $180^\circ$  out of phase with the current flow in diode 4.

Once the four individual diode currents have been determined, they can be combined to form the IF outputs at  $I_1$  and  $I_2$ . The current exiting  $I_1$  can be written as:

$$i_{11} = i_1 - i_2 + j i_3 - j i_4 \\ = K [(-1)^m - 1 + j^{(1-m)}(-1)^n - j^{(1+m)}(-1)^n]$$

Currents  $i_3$  and  $i_4$  are multiplied by  $j$  because of the  $90^\circ$  phase shift in the IF quadrature coupler. Currents  $i_2$  and  $i_4$  are negative because they are entering (instead of exiting) at the node connecting the diodes to the IF coupler. Similarly, the current exiting  $I_2$  can be written as:

$$i_{12} = j i_1 - j i_2 + i_3 - i_4 \\ = K [j(-1)^m - j + j^m(-1)^n - j^m(-1)^n]$$

Table 1 summarizes which mixing products exit at  $I_1$  and  $I_2$ . Notice first that the R±L products exit at  $I_1$ , and the L-R product exits at  $I_2$ . Also, notice that every other odd product exits  $I_1$  and  $I_2$ . Mixing products ( $\pm L+R$ ), ( $\pm L+5R$ ), ( $\pm L+9R$ ), etc. and (L-3R), (L-7R), (L-11R), all exit at  $I_1$ . And mixing products

(L-R), (L-5R), (L-9R), etc., and (L+3R), (L+7R), etc., all exit at  $I_2$ . Finally, notice that the products exiting at  $I_1$  and  $I_2$  are always in quadrature with each other. When analyzing IM suppression, the bandwidth of the output port must be considered because many of the products in Table 1 could be outside the frequency range of the low-pass IF output.

The preceding analysis can be used to quickly analyze mixer/quadrature-hybrid networks to determine which products will exit the mixer ports. The phase angle of each diode current can be written in its final form in terms of  $j^m$ ,  $j^n$ ,  $(-1)^m$ ,  $(-1)^n$  by inspection, and then summed. A simple table of output products can then be written.

### APPENDIX B: IMAGE REJECTION AS A FUNCTION OF AMPLITUDE AND PHASE MATCH

This analysis shows the relationship between image rejection and amplitude phase imbalances [6]. Image rejection is defined as the ratio of the magnitude of the image signal and the desired signal. Therefore, the image rejection at  $I_1$  in Figure 2 is:

$$I_R = -20 \log \frac{|I_1(m=+1)|}{|I_1(m=-1)|} \quad (3)$$

From equation (1)  $|I_1'| = |I_2'| = I_{mn}$ ; using this and rewriting equation (1), we obtain:

$$I_1 = 1/2 [|I_1'| \angle \theta_1 + |I_2'| \angle (\theta_2 + (m+1) 90^\circ)] \quad (4)$$

From equation (4) we obtain the following equations for  $I_1(m=+1)$  and  $I_1(m=-1)$ :

$$I_1(m=+1) = 1/2 (|I_1'| \angle \theta_1 - |I_2'| \angle \theta_2) \quad (5A)$$

$$I_1(m=-1) = 1/2 (|I_1'| \angle \theta_1 + |I_2'| \angle \theta_2) \quad (5B)$$

For practical applications,  $I_1'$  and  $I_2'$  are not exactly amplitude and phase matched. If an amplitude imbalance factor of  $A$  and a phase imbalance factor of  $\theta$  are included in equations (5A) and (5B), we obtain:

$$I_1(m=+1) = 1/2 [|I_1'| \angle \theta_1] [(1 - A \angle \theta_2)] \quad (6A)$$

$$I_1(m=-1) = 1/2 [|I_1'| \angle \theta_1] [(1 + A \angle \theta_2)] \quad (6B)$$

Where,

$$A = \left| \frac{I_2'}{I_1'} \right| \text{ and } \theta = \theta_2 - \theta_1$$

The factor  $A$  is equal to the sum of the individual amplitude imbalances in the rf and IF hybrids and the two mixers. The factor,  $\theta$ , is

Mixing Product	(RF) m	(LO) n	$I_1$	$I_2$
R+L	1	1	$K[-1-1-1-1] = -4K$	$K[-j-j+j+j] = 0$
R-L	1	-1	$K[-1-1-1-1] = -4K$	$K[-j-j+j+j] = 0$
L-R	-1	1	$K[-1-1+1+1] = 0$	$K[-j-j-j-j] = -4jK$
L to I isolation	0	1	$K[-1-1-j+j] = 0$	$K[j-j-1+1] = 0$
R to I isolation	1	0	$K[-1-1+1+1] = 0$	$K[j-j-j-j] = -4jK$
2x1	2	1	$K[1-1+j+j] = 0$	$K[j-j+1-1] = 0$
-2x1	-2	1	$K[1-1+j-j] = 0$	$K[j-j+1-1] = 0$
3x1	3	1	$K[1-1+1+1] = 0$	$K[j-j-j-j] = -4jK$
-3x1	-3	1	$K[1-1-1-1] = -4K$	$K[j-j+j+j] = 0$
5x1	5	1	$K[-1-1-1-1] = -4K$	$K[-j-j+j+j] = 0$
-5x1	-5	1	$K[-1-1+1+1] = 0$	$K[j-j-j-j] = -4jK$

Table 1. Summary of which mixing products exit  $I_1$  and  $I_2$ .



the total phase imbalance which is due to the sum of the deviation from quadrature in the rf and IF hybrids, and the phase imbalance of the two mixers.

A and θ also include the effects of hybrid directivity and impedance mismatches between the hybrids and the mixers. Imperfect hybrid directivity causes additional phase errors, and impedance mismatches cause amplitude ripple [7].

Substituting equations (5A) and (5B) into (3) results in the following equation for image rejection as a function of A and θ:

$$IR = -10 \log \left( \frac{1 + A^2 - 2A \cos\theta}{1 + A^2 + 2A \cos\theta} \right)$$

$$A = 10^{-\left(\frac{A \text{ dB}}{20}\right)}$$

The effect of A and θ on image rejection is illustrated in Figure 8 [6].

Example: If the if hybrid amplitude imbalance is +0.5 dB, the IF hybrid amplitude imbalance is +0.5 dB, and the mixer amplitude match is -0.5 dB, the total amplitude imbalance is 0.5 + 0.5 - 0.5 = 0.5 dB. If the total phase imbalance is 10 degrees, the image rejection is 20.7 dB. This estimate of the image rejection is optimistic, since it does not include the effects of VSWR and imperfect hybrid directivity.

### CONCLUSION

In summary, image-reject and single' sideband mixers provide a valuable means of solving difficult system problems posed by conventional double-balanced mixers. Using phase cancellation instead of filtering for image rejection and sideband suppression, fewer expensive components, such as mixers, VCOs, and amplifiers are required. This means that reliability is increased and cost is reduced. The theory and operation of IRMs and SSMs has been discussed, and key parameters have been defined. The tradeoffs between sideband, inter-modulation and carrier suppression for upconverter applications are outlined, and practical design guidelines

given. IRMs and SSMs are increasingly solving key system problems. It is the authors' intent that this article help further this progress.

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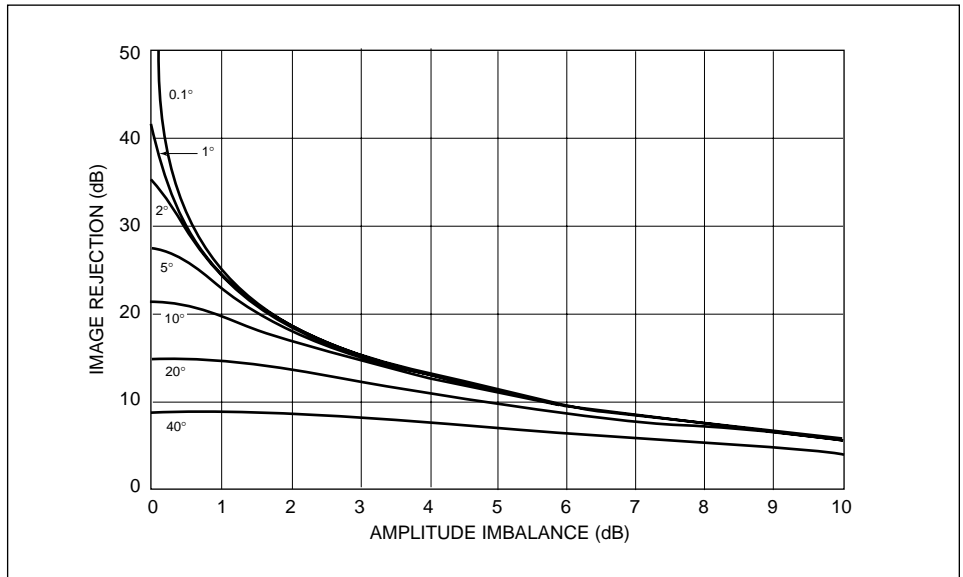


Figure 8. Image rejection vs. amplitude and phase imbalance.