Rigorous Analytical Expressions for the Effective Dielectric Constants of the Shielded Symmetrical Bandline

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ABSTRACT

This article is a continuity of the reference [1] and it presents a set of accurate closed-forms formulas for the effective dielectric constants of the shielded symmetrical bandline. This formulas are based on rigorous analysis by finite element method (FEM) [2], method of moment (MoM) [3] and curves fitting techniques.

The good coherence of the two numerical methods (FEM and MoM) [1] allows to generate rigorous analytical solutions for a wide-range of discontinuity angles and are suitable for all shielded symmetrical bandlines which have an outer-inner conductors radius ratio between 2 and 6.

These expressions can be easily implemented in CAD simulation tools, to design many components as RF resonators, RF couplers [1], filters, transmission lines,… for wireless communication and probes for material characterization [4].

INTRODUCTION

The electrical properties of a lossless shielded symmetrical bandline with a quasi-TEM-mode can be described in terms of even (Z_{oe}, ε_{effe}) and odd (Z_{oo}, ε_{effo}) mode impedances and effective dielectric constants, and its primary parameters [L] and [C].

A variety of numerical techniques are available to accurately determine the characteristic impedance, the effective dielectric constant and the primary parameters of the shielded symmetrical bandline. But they are time-consuming and too tedious for use in circuit design, where closed-form analytical models are to be preferred. By applying FEM and MoM analyses along with curve-fitting strategies, it is possible to develop these closed-form expressions for determining the characteristic impedance, the effective dielectric constant and primary parameters of the shielded symmetrical bandline.

In [1], a set of closed-form equations was developed to determine the characteristic impedances and the primary inductance and capacitance matrices (the [L] and [C] matrices, respectively). In order to complete the study, we present rigorous analytical expressions for the effective dielectric constants of the shielded symmetrical bandline having an outer-inner conductors radius ratio between 2 and 6.

SHIELDED SYMMETRICAL BAND LINE

The line is assumed to be lossless with inner conductors of radius r_o, negligible thickness w, a discontinuity angle θ and an outer shield of radius r_b. Dielectric materials with permittivities ε_{r1} and ε_{r2} are placed respectively inside the bands and between the bands and the shield.

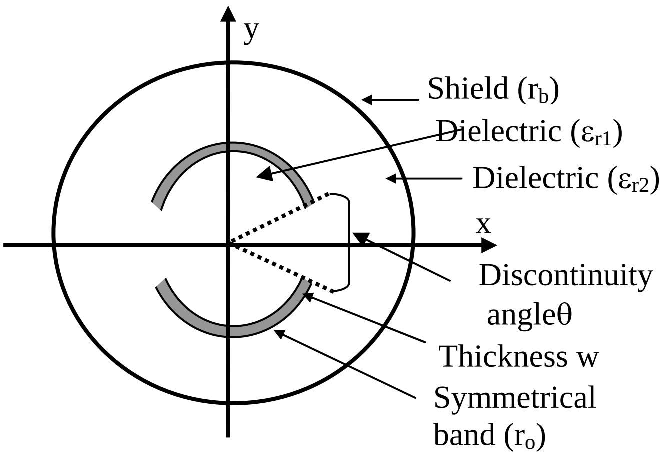

Figure 1 : Cross section of the shielded symmetrical bandline.

NUMERICAL RESULTS

The numerical results for the effective dielectric constant of the shielded symmetrical bandline using the FEM and MoM methods are shown in figures 2 to 4. These results demonstrate the excellent coherence between the FEM and MoM methods.

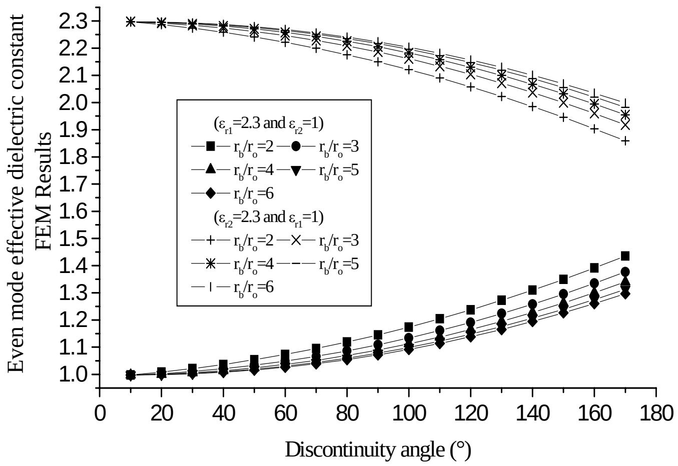

Figure 2 : Effect of the discontinuity angle on the even mode effective dielectric constant using

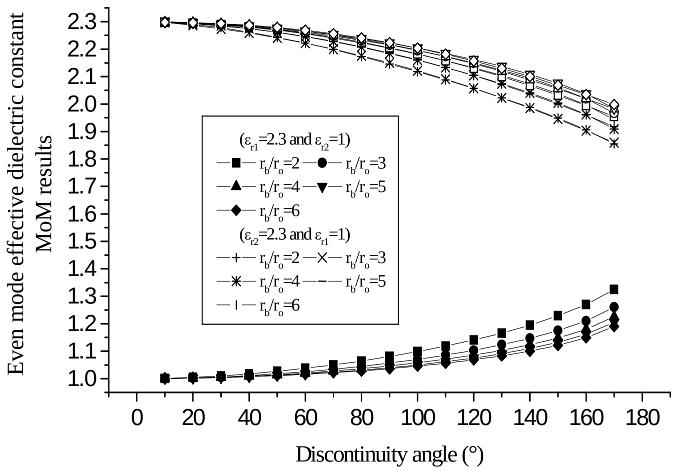

Figure 3 : Effect of the discontinuity angle on the even mode effective dielectric constant using MoM.

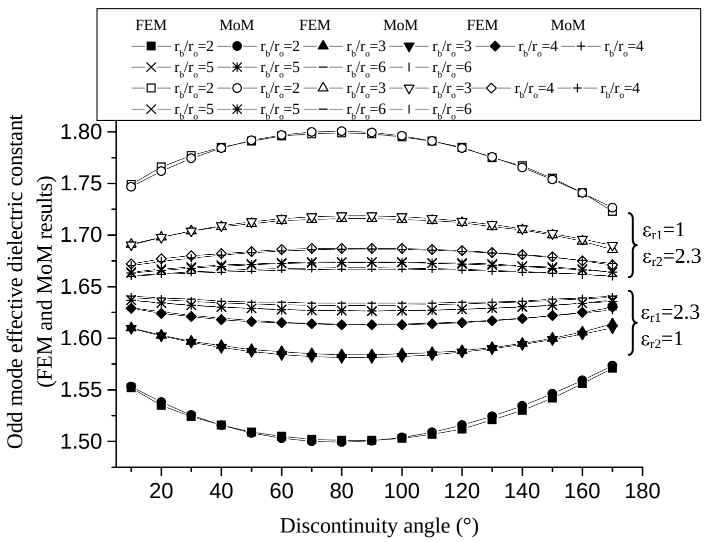

Figure 3 : Effect of the discontinuity angle on the odd mode effective dielectric constant.

DERIVATION OF ANALYTICAL MODELS

1.EVEN MODE EFFECTIVE DIELECTRIC CONSTANT

The even mode effective dielectric constant (ε_{effe}) of the shielded symmetrical bandline can be expressed by the relations (1) and (2) for $2 \le r \le 6$ *and* $0 < \theta < 180°$.

- For $\varepsilon_{r2} / \varepsilon_{r1} \ge 1$

$$\varepsilon_{effe} = \varepsilon_{r1}\left(\varepsilon_{eff1} + a_o\left(b_0 - 2.3\right)\right) \qquad (1)$$

- For $\varepsilon_{r2} / \varepsilon_{r1} < 1$

$$\varepsilon_{effe} = \varepsilon_{r1} + \varepsilon_{r2}\left(1 - \varepsilon_{eff1} - a_o\left(\frac{1}{b_0} - 2.3\right)\right) \qquad (2)$$

Where:

$a_o = 1.01474 - 0.00126\,\theta - 5.978\,10^{-6}\,\theta^2$

$b_o = \varepsilon_{r2} / \varepsilon_{r1}$

$\varepsilon_{eff1} = \varepsilon_{eff*} + b_1\theta + b_2\theta^2$

$\varepsilon_{eff*} = 2.3 + 0.01061 e^{-(r-2)/1.53801}$

$b_1 = 3.0074\,10^{-4} - 0.00103\, e^{-(r-2)/1.27842}$

$b_2 = -4.11425\,10^{-6} - 5.62897\,10^{-6} r + 1.22252\,10^{-6} r^2$
$- 8.29407\,10^{-8} r^3$

$r = r_b / r_o$

2.ODD MODE EFFECTIVE DIELECTRIC CONSTANT

For $2 \le r \le 6$ *and* $0 < \theta < 180°$ the odd mode effective dielectric constant (ε_{effo}) is expressed by the relations (1) and (2), where:

$a_o = 0.51371 + 3.53\,10^{-3}\,\theta - 5.3888\,10^{-5}\,\theta^2$
$+ 3.53407\,10^{-7}\,\theta^3 - 9.07707\,10^{-10}\,\theta^4$

$b_o = \varepsilon_{r2} / \varepsilon_{r1}$

$\varepsilon_{eff1} = \varepsilon_{eff*} + b_1\theta + b_2\theta^2 + b_3\theta^3$

$\varepsilon_{eff*} = 1.65706 + 0.07192\, e^{-(r-2)/0.94195}$

$b_1 = 2.09284\,10^{-4} + 0.00171 e^{-(r-2)/1.12538}$

$b_2 = -1.5713\,10^{-6} - 1.2475\, e^{-(r-2)/1.00823}$

$b_3 = 1.9738\,10^{-9} + 1.37349\,10^{-8}\, e^{-(r-2)/0.90595}$

$r = r_b / r_o$

The relative error between the numerical and the analytical results are less than 2% in a wide range, indicating the good accuracy of the proposed expressions for the shielded symmetrical bandline.

CONCLUSION

This article presents a set of accurate closed-form formulas for the dielectric constants (ε_{effe}, ε_{effo}) of the even and odd modes of the shielded symmetrical bandlines.

The expressions obtained from the finite element method and the moments method, are valid in a wide range of the discontinuity angle and the outer-inner conductors radius ratio.

REFERENCES

1. N. Ben Ahmed and M. Feham, Analyzing EM parameters for shielded bandline, Microwaves & RF, March 2006, pp.86-92.
2. N. Ben Ahmed and M. Feham, Finite Element Analysis of RF couplers with Sliced Coaxial Cable, Microwave Journal, Vol.2 N°2, 2000, pp.106-120.
3. A.R. Djordjevic, D.Darco, M.C. Goran, T.K. Sarkan, Circuit Analysis Models for Multiconductors Transmission Lines, Artech Housse, 1997.
4. N. Ben Ahmed and M. Feham, Design NMR probes at high frequencies, Microwaves & RF, Vol. 41, No. 2, 2002, pp.77-103.